Design Method for Piezoelectric Bending Generators in Energy Harvesting Systems

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ABSTRACT

The increased demand for mobile systems using low-power electronics leads to a need for new power sources. Using batteries as power source may be inapplicable in distributed systems like wireless sensor networks because the batteries have to be exchanged frequently. Energy Harvesting systems are one possible energy source for such systems exploiting environmental energy like mechanical vibrations. One good solution to convert vibration energy is the use of piezoelectric generators usually realised as piezoelectric bending beams.

The generators convert mechanical energy to electrical energy due to resulting strain of the element. However, the power output of piezoelectric generators is a challenging task even if low-power applications have to be driven. Due to the low electric power output of piezoelectric generators, it is an important task to obtain a suitable geometric design of the transducer element. Beside the element dimensions the electric power output depends on the input excitation as well as on the electric load to be powered.

To analyse the system behaviour, input variables and the generator itself have to be described in a mathematical model. This enables the calculation of optimal elements in principle. A modal electro-mechanical model of the piezoelectric element assuming to be base-excited is used in this paper. Although the modal model is very helpful to analyse the system, it cannot be easy used to determine a proper design of the piezoelectric elements. The problem is that the parameters of the model do not show any apparent relations to geometric dimensions or material data. Therefore, a mathematical method to obtain the parameters from the physical properties of a piezoelectric bending element is briefly described. The knowledge of the link between physical and modal parameters allows the usage of the mathematical model as a qualified design method. The input parameters of the linked model are the material data which can be found on data sheets. Additionally, boundary conditions of the environment like the impedance of the driven load and the vibration excitation has to be specified. The linked model shows the influences on power output to connected electric loads. The given power demands of applications which have to be satisfied yields in a design space of suitable elements. The design method enables the development engineer to select piezoelectric generator elements.

Keywords: Energy Harvesting, Piezoelectric Element, Energy Supply, Autonomous System

1. INTRODUCTION

Energy harvesting systems get more and more in the focus of research. A very promising method to harvest energy from the system surroundings is the use of piezoelectric transducers. In mechanical structures the use of the piezoelectric effect is an appropriate solution to convert ambient vibration energy to electric energy. Utilising this effect, usually a bending beam structure is used, as for example by Roundy[1] or Sodano.[2]

The design of piezoelectric energy harvesting systems depends on the surrounding kinetic energy which excites the harvester as well as on the connected electrical application which has to be powered. Therefore, it is necessary to have the ability to investigate the system's behaviour to understand the influences of internal and external parameters. The understanding of the interdependencies will allow the creation of a design guideline.

A suitable modal model is described in this paper that is extended by geometric dimensions. The extension describes the modal parameters by geometric and material data following the approach of Lenk[3] instead of

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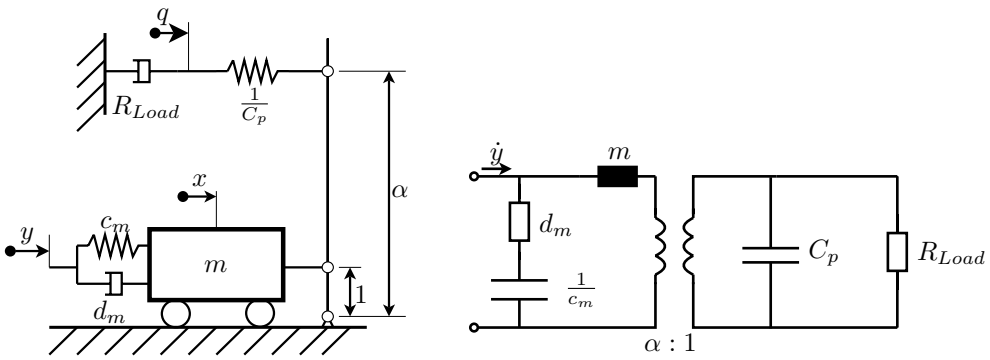

(a) Mechanical Model (b) Electrical Model

Figure 1. Equivalent models for base excited piezoelectric energy harvesting systems.

identifying the lumped parameters by measurements. The geometric model is improved by considering the passive layer for the electric system behaviour.

Using the modal model, the influence of the resistive load on the power output is investigated in detail. Therefore, a way to describe the admittance of the loaded system is explained. The analysis of the loaded admittance shows the influence of the resistance on the working frequency. The knowledge of the behaviour of the loaded system is used to analyse the influence of geometric dimensions on the power drain over the load resistance. The investigation of these effects helps to create a design guideline which is shown in this paper for an exemplary power demand.

2. MODELLING OF PIEZOELECTRIC BIMORPHS

A piezoelectric element is an electro-mechanic system. To describe the behaviour of the investigated system, electric as well as mechanic domains have to be joined in one model. The system's behaviour on the electric and on the mechanic side is first modelled using a modal model that allows an appropriate system analysis. In a further step it is briefly explained how geometric and material parameters can be used to derive the lumped parameters of the modal model.

2.1. MODAL MODEL FOR A PIEZOELECTRIC ELEMENT

A suitable linear model using lumped parameters is presented in this section assuming a base excited system. The model combines mechanical and electric properties using an electro-mechanic analogy where mechanic force F is represented by electric voltage u and mechanic displacement x commensurates to electric charge q. Due to this analogy, a mass m can be described by an inductance L, a damper d is analogue to a resistance R and a spring c is considered as a capacitance $\frac{1}{C}$.

Utilising this analogy yields to a single degree of freedom system which can be written as mechanical or electrical equivalent model as shown in Fig. 1. Both representations have a mechanic and a electric part combined by a transducer element. Examining the mechanical model in Fig. 1(a), the lower part shows the properties of the mechanic oscillator whereas the upper part describes the electric parameters capacitance as well as the electric load modelling the connected application to be powered. The two domains are coupled by a lever of length α to indicate the transmission factor. The electric energy output is modelled as power drain on the electric load R_{Load}. In the electrical model Fig. 1(b), both domains coupled by an ideal transformer which is the electric equivalent for a lever. Two boundary conditions has to be considered utilising one of the models: First, the system has to be investigated to to its eigenfrequency and second the input velocity $\dot{x}$ has to be small compared to the tip velocity $\dot{y}$. Any dielectric losses are considered to be neglectable.

The system's behaviour can be mathematically formulated using the equations of motion of this system. Transferring these into the Laplace space they can be solved to the voltage u depending on the frequency shown in Equation 1 where the input parameter $\dot{y}$ is renamed to v_{Base}. According to Ohm's law, the total power output

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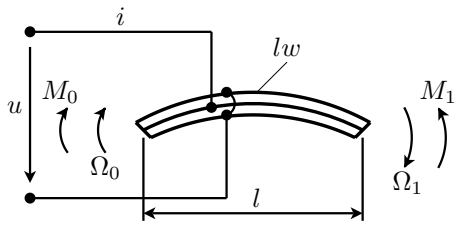

Figure 2. Design of a piezoelectric bimorph neglecting the adhesive layer in the middle of the beam.

for an application can be determined by Equation 2. Replacing the Laplace variable s by their equivalent $(j\ \omega)$ yields to a real value for $P(\omega)$ due to the conjugate complex term for the current $i = \frac{U_L}{R_{Load}}$. Identification of a correct parameter set for the the five lumped system parameters c_m, C_p, d_m, m and α enables the determination of the power drain on the connected load. The identification can be done by measurements admittance frequency plots of the system very well.[4]

$$U_L(s) = \frac{\frac{1}{C_p}\ (c_m + d_m\ s)\alpha\ R_{Load}}{s\ \frac{1}{C_p}\ R_{Load}\ \alpha^2 + (c_m + s\,(d_m + m_1\ s))\left(\frac{1}{C_p} + s\ R_{Load}\right)} v_{Base} \quad (1)$$

$$P(s) = U_L(s)\left(\frac{U_L(s)}{R_{Load}}\right)^* \quad (2)$$

2.2. GEOMETRIC MODEL FOR A PIEZOELECTRIC ELEMENT

Another possibility to obtain the modal system's parameter is the identification using measurements. These parameters utilising geometric and material properties are described according to Lenk[3] in this section. The geometric parameters are length l, width w and thickness h of the beam. Material properties of the piezoelectric element are density ρ, compliance s_{11}^E, relative dielectric constant ϵ_{33}^T, piezoelectric constant d_{31} and quality factor Q_m.

The possibility to use a geometric model allows the analysis of the behaviour of the piezoelectric element according to its dimensions. To take geometric data into account, the system has to be described in more detail regarding to the system design as a piezoelectric bending beam as shown in Fig.2. In a first step, the passive layer of the piezoelectric bending element is neglected in the model.

The continuous, dynamic behaviour of a bending beam can be described by equations of motion and depends basically on the total mass m_b and compliance s_b of the beam; both depending on geometric and material properties. In general, forces F and velocities v as well as moments M and angular velocities Ω can occur on the beam's end. For an energy harvesting application, the beam is clamped on on side and has a free end on the other side. Therefore, the boundary conditions are: No moment on the free side and no angular velocity at the clamped side. Considering this boundary condition, the equation of motions can be written according to Euler-Bernoulli separating time and location dependent expressions. The Rayleigh functions (Eqns. 5 to 8) that depends onη are used to solve the equations of motion. This results in the mathematic vector system shown in Equation 9.[5]

$$\eta = \sqrt{\frac{\omega}{\omega_0}} \quad (3)$$

$$z_0 = \sqrt{m_b}\ s_b \quad (4)$$

$$S_R(\eta) = 0.5(sinh\eta + sin\eta) \quad (5)$$

$$s_R(\eta) = 0.5(sinh\eta - sin\eta) \quad (6)$$

$$C_R(\eta) = 0.5(cosh\eta + cos\eta) \quad (7)$$

$$c_R(\eta) = 0.5(cosh\eta - cos\eta) \quad (8)$$

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$$\begin{pmatrix} v_0 \\ F_0 \end{pmatrix} = \begin{pmatrix} \frac{l}{\eta}\left(\frac{C_R^2 - s_R S_R}{s_R}\right) & \frac{1}{jz_0 l}\left(\frac{c_R s_R - C_R S_R}{s_R}\right) \\ jz_0 l\left(\frac{S_R C_R - s_R c_R}{s_R}\right) & \frac{\eta}{l}\left(\frac{s_R^2 - S_R^2}{s_R}\right) \end{pmatrix} \begin{pmatrix} \Omega_1 \\ M_1 \end{pmatrix} \tag{9}$$

Next, the piezoelectric effect depending on material strain and electric displacement has to be included to this mathematical model. The well known linear material law for piezoelectric materials for 31 direction can be formulated as:

$$T_1 = \frac{1}{s_{11}^E(1-k_{31}^2)} S_1 - \frac{1}{d_{31}}\left(\frac{k_{31}^2}{1-k_{31}^2}\right) D_3 \tag{10}$$

$$E_3 = -\frac{1}{d_{31}}\left(\frac{k_{31}^2}{1-k_{31}^2}\right) S_1 + \frac{1}{\epsilon_{33}^T(1-k_{31}^2)} D_3 \tag{11}$$

$$k_{31}^2 = \frac{d_{31}^2}{e_{33}^T s_{11}^E} \tag{12}$$

Considering the system piezoelectric bending beam with two piezoelectric layers as shown in Fig. 2, we can find a relationship between current i, voltage u, moment M and angular velocity Ω. This relationship can be used, to combine the piezoelectric effect with the continuous beam model as shown in Equation 13. Using this mathematical model allows the determination of the modal parameters. The equations are shown in Table 1.

$$\begin{pmatrix} v_0 \\ F_0 \end{pmatrix} = \begin{pmatrix} \frac{l}{\eta}\left(\frac{C_R^2 - s_R S_R}{s_R}\right) & \frac{1}{jz_0 l}\left(\frac{c_R s_R - C_R S_R}{s_R}\right) \\ jz_0 l\left(\frac{S_R C_R - s_R c_R}{s_R}\right) & \frac{\eta}{l}\left(\frac{s_R^2 - S_R^2}{s_R}\right) \end{pmatrix} \cdot \begin{pmatrix} j\ \omega\ C_p & \alpha' \\ \alpha' & -\frac{1}{j\ \omega\ s_b} \end{pmatrix} \begin{pmatrix} u \\ i \end{pmatrix} \tag{13}$$

$$C_p = 4\epsilon_{33}^T\ (1-k_{31}^2)\frac{l\ w}{h} \tag{14}$$

$$\alpha' = \sqrt{\frac{Cp}{s_b}\frac{0.75k_{31}^2}{(1-0.75k_{31}^2)}} \tag{15}$$

Table 1. Mathematical Determination of the modal model parameters using geometric and material data.

Modal Parameter	Mathematical Description
m	$\frac{\rho\ l\ w\ h}{4}$
c_m	$\frac{w\ h^3}{3.884\ l\ s_{11}^E\ k_1}$
d_m	$\frac{w\ h^2}{4\ l\ Q_m}\sqrt{\frac{\rho}{0.971 s_{11}^E\ k_1}}$
α	$\frac{w\ h}{0.727\ l}\frac{3\ k_2\ s_{11}^E}{\epsilon_{33}^T}$
C_p	$4\ \epsilon_{33}^T\frac{l}{h}\frac{w}{h}k_1$
k_1	$\frac{1-k_{31}^2}{1-0.75\ k_{31}^2}$
k_2	$\frac{1-0.75\ k_{31}^2}{0.75\ k_{31}^2}$

Here a piezoelectric bimorph that consists of the material VIBRIT 1100 is used. The geometric and material properties are shown in Table 2. For a first comparison between model and the real system it has to be stated that the model does not consider a substrate layer. Due to this neglection the value of C_p is calculated to low. The underestimation results due to the incorrect assumption of the thickness of the piezoceramic layers that

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concludes to less capacitance per meter in the piezoceramic layers. A first approach to include this effect of the adhesive layer to the piezoelectric capacitance is the use of an passive height $h_{passive}$. The adhesive layer of this height is considered to be perfect conducting so that it does not affect the electric behaviour. To determine the capacitance C_p, the thickness of the passive layer has to be subtracted from the total thickness as shown in Equation 16. Due to the conducting middle layer, we do not have a constant electric field over the total height of the beam which is a necessary assumption in the material law. Therefore, the dielectric constant ϵ_{33}^T is modified. This leads to a system behaviour as if the complete thickness has the same dielectric property. Because the transmission factor α does depend on this dielectric parameter, it is modified according Equation 17.

Table 2. Geometric and material data of used piezoelectric bimorph of material VIBRIT 1100.

l	w	h	$h_{passive}$	ρ	k_{31}	s_{11}^E	ϵ_{33}^T	Q_m
$39.5mm$	$7.2mm$	$0.8mm$	$0.3mm$	$8100\frac{kg}{m^3}$	0.42	$14.2 \cdot 10^{-12}\frac{m^2}{N}$	$39.825\frac{nF}{m}$	50

$$C_p = 4\ \epsilon_{33}^T \frac{l\ w}{h - h_{passive}} k_1 \tag{16}$$

$$\alpha = \frac{w\ h}{0.727\ l} \frac{3\ k_2\ s_{11}^E}{\epsilon_{33}^T\ \frac{h-h_{passiv}}{h}} \tag{17}$$

3. MODEL ANALYSIS

The arising power drain on the resistor is one of the most important questions for a model analysis. If this demand can be calculated for different excitations in amplitude and frequency as well as varying load resistances, piezoelectric generators can be designed. The necessary or even optimal size of piezoelectric bimorphs can be calculated due to the possibility to describe the modal parameters by geometric properties.

3.1. INFLUENCE OF ELECTRIC LOAD

One important factor of the power output of a piezoelectric energy harvesting system is the connected load. To analyse the behaviour regarding the connected resistance, the modal model is used. To compare the results to a real system, the lumped parameter set is obtained by identification measurements of the admittance frequency plot as described by Littmann.[4] Therefore, a the admittance of a bimorph was measured for different excitation levels. The resulting plots are shown in Fig. 3 showing amplitude and phase regarding to the frequency. The identified parameter sets according to this plots are shown in ascending excitation in Table 3. In the following figures these parameter sets are used plotted as solid, dotted or dashed line as indicated in the table.

Table 3. Identified Parameter Sets for different excitation levels.

Excitation level	$m\ 10^{-3}\ [g]$	$d_m\ 10^{-3}\ [\frac{Ns}{m}]$	$c_m\ [\frac{N}{m}]$	$\alpha\ 10^{-3}\ [\frac{N}{V}]$	$Cp\ 10^{-9}\ [F]$	$f_r[Hz]$	Q_m
low (line)	346.1	10.878	774.3	2.427	84	238	47.6
mediate (dashed)	305.0	17.250	653.8	2.388	81.6	233	25.9
high (dotted)	288.8	28.879	526.0	2.2443	85.37	222	13.0

Regarding to the maximum power transfer theory, the maximum power output on the connected electric load can only be achieved if the output impedance of the piezoelectric element equals the input impedance of the load. As described by Twiefel et al.,[6] the frequency behaviour of the output impedance is affected by the connected load. To consider this effect, the impedance of the total system is calculated. Therefore, an additional electric output over the resistor is imagined to get a four-pole system as shown in Fig. 4 as electric equivalent circuit.

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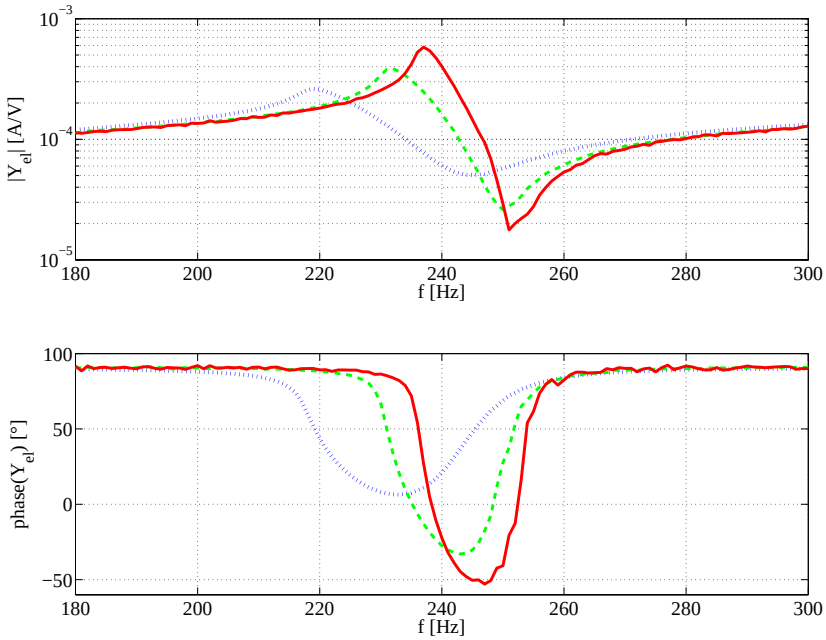

Figure 3. Frequency Response Y_{el} of a piezoelectric element with different electric excitation amplitudes.

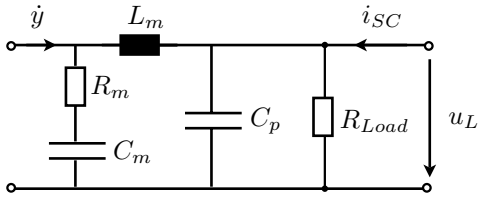

Figure 4. Electric equivalent of the loaded piezoelectric element with additional output for admittance determination.

On the new output current and voltage can be determined. The equation for the loaded output impedance Z_L over the frequency is calculated by the ratio between the voltage over the connected load U_L (see Equation 1) to the short circuit current i_{SC}. Due to the parallel circuit structure, the equation of the admittance Y_L is simply the addition of the susceptance of the resistor to the admittance Y of the unloaded system.

$$i_{SC} = \frac{\alpha(d_m\ s + c_m)}{m\ s + d_m + \frac{c_m}{s}} v_{base} \tag{18}$$

$$Z_L = \frac{U_L}{i_{SC}} = \frac{R_{Load}(c_m + s(d_m + m\ s))}{(1 + C_p\ R_{Load}\ s)(c_m + s(d_m + m\ s)) + R_{Load}\ s\ \alpha^2} \tag{19}$$

$$Y_L = \frac{1}{R_{Load}} + s(C_p + \frac{\alpha^2}{c_m + s(d_m + m\ s)}) \tag{20}$$

The power drain on the resistance $P(s)$ can be calculated in general by Equation 2. To obtain an equation to determine the working frequency for optimal power output, the characteristic function Y_L is investigated. Now

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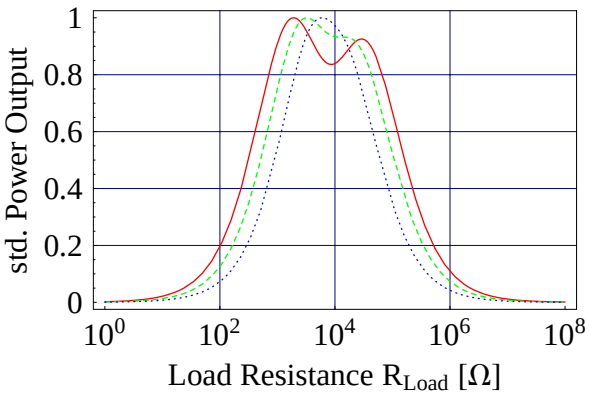

Figure 5. Modelled power output over load resistance. For each combination an optimal working frequency regarding frequency is assumed. The different curves indicates different excitation levels. The parameter set has been identified using measurements.

it has to be remarked, that Y_L is only a virtual output. In case of resonance of the investigated system, most current has to flow through the connected load. That implies that there is no current flow over the imagined output. So the case of resonance occurs if Equation 21 holds. Solving this term to s_{f_r} the optimum resonance frequency for resistive loads to maximize the power output is obtained.

$$Y_L(s, R_{Load}) = 0 \tag{21}$$

The yielded equation for the optimum frequency can be included in the Equation 2 for $P(s)$. The resulting expression depends on the electric load only and is visualised in Figure 5. This plot shows the dependency of the resistance to the power output for different parameter sets shown in Table 3. Each point indicates an optimal working frequency.

Small excitation levels shows two local maxima whereas higher levels results in only one global maximum. The two maxima can be found at resonance and at antiresonance. Due to the low current flow in case of antiresonance, the optimal load resistance to get most power drain has to rise. This behaviour is shown in Figure 6 for the different identified parameter sets. The dots indicates the optimal resistances. It can be seen that low excited systems have the two best resistances nearly at the inflexion points. The optimum point for higher excited systems moves the curve upwards in case of the resonance frequency.

This can be explained by the increasing damping characteristics of the piezoelectric element. If damping is high enough, frequency response plots of the admittance does not have a zero crossing of the phase. Lower damped piezoelectric elements crosses the zero line two times: In resonance and in antiresonance frequency. This can be seen in Figure 7 where the phases for different excitations are shown. For each parameter set three lines are pictured. The light line shows the unloaded system, the fat lines of each colour show the admittances of the whole system for the minimal and maximal optimum (very fat line) resistance. In this picture, dots indicates the best possible working frequency as before. Examining the plots, it can be recognized that low damped systems show the same optimal point for all three curves. The higher the system damping gets, the more the dashed curve moves away from the optimum point.

3.2. INFLUENCE OF GEOMETRY

Replacing the modal parameters as explained in Section 2.2 enables the analysis depending on the dimensions of the beam. If the behaviour of the system regarding geometrical properties is known, a design method can be derived. The important factors for the design process are on the one hand the external parameters excitation frequency and excitation level as well as electric load. On the other hand, the geometric parameters length,

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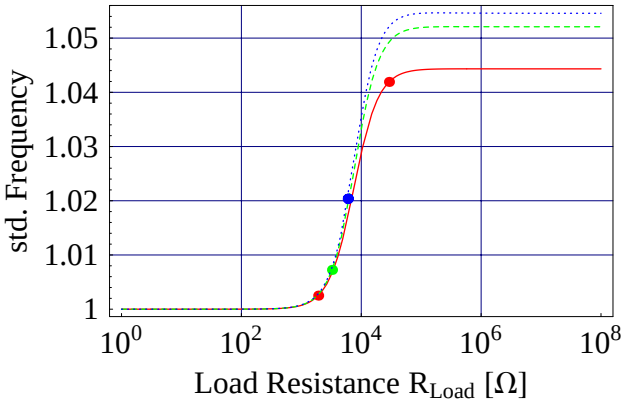

Figure 6. Dependency of load resistance and resonance frequency. It can be seen that the optimal load resistance shifts the working frequency in defined circumstances. We obtain a stable level for the minimum and the maximum working frequency.

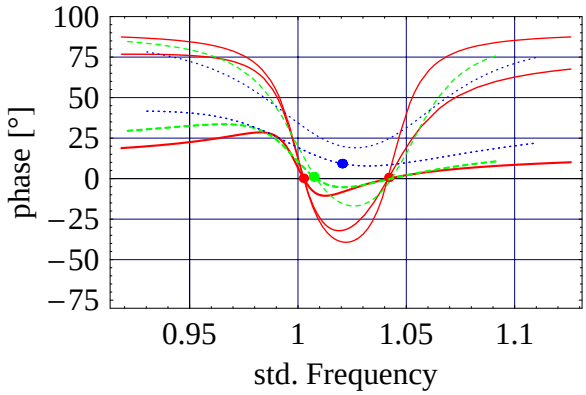

Figure 7. Admittances for loaded and unloaded systems for different excitation levels. Each colour represents one excitation level. The dashed line shows the unloaded system.

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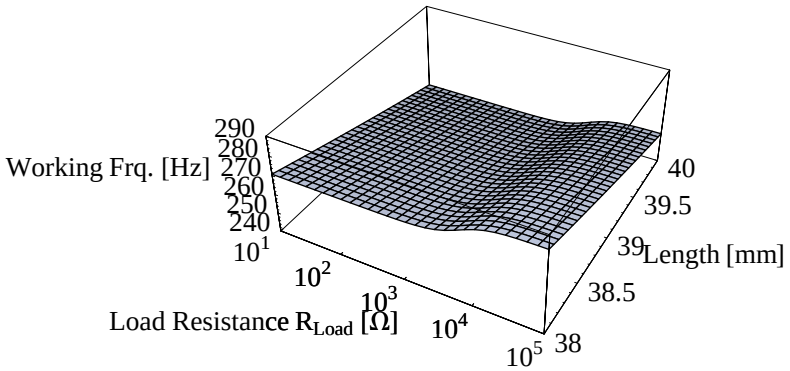

Figure 8. Dependency of load resistance, length and resonance frequency. A similar effect according to Fig. 6 can be observed.

width and thickness of the piezoelectric bimorph are of interest. The beam's thickness is held constant in the following analysis.

For the analysis of the geometric factors the material and geometric data of Table. 2 are used. The excitation level is held constant at $v_{base} = 15mm/s$ at an excitation frequency of 250 Hz matching the eigenfrequency of the investigated system. Examining Equation 1, the voltage and power output is linear to the excitation level. Therefore, the analysis methods did not investigate different levels, because changing excitation levels do not change the systems behaviour in general.

One important factor to obtain as much power as possible is the resonance frequency of the system. It depends mainly on the thickness and length of the beam. But as stated before, the resistance itself has an influence on the optimal working frequency. Plotting the resistance on which the maximum power is drained and the beam length to the frequency (Fig. 8) shows that the load resistance shifts the working frequency from the resonance to the antiresonance frequency for defined conditions.

Altering width and length shows the behaviour regarding power output as depicted in Fig. 9. For each of the three diagrams, the resistance R_{Load} is constant. The resistances are selected for three values showing a different case as can be seen in Fig. 8. The lowest resistance represents a case where the system operates in its resonance frequency, the highest resistance is selected for a system operation in antiresonance. The electric load of 3200Ω as shown in the middle is selected out of the range for an operation neither in resonance nor in antiresonance.

As shown before, it is important to consider the interdependencies of the external variables to the geometric dimensions of the piezoelectric element to design a suitable bimorph. Usually, the excitation and the impedance of the electric application is known. Knowing excitation and resistance allows the creation of a figure as shown in Fig. 9. Utilising this figure, power drain over the load is yielded for every combination of the length and width of the element. To gain information if a combination is power optimal, a line is included into the diagram as can also be seen in Fig. 9. The dashed line is created by calculating if the power output is a local maximum of the function. To get the information of the maximum, the power function Equation 2 is derived to ω. The roots of the derived function $\frac{d}{d\omega}P(\omega)$ specify the local maxima. This information is used to obtain the appropriate length for each width. Therefore, it is evident that there is no other point in the diagram for a given width on which more power output can be expected.

If an element is designed, it is often not necessary to work in an optimum working point. It is most important to harvest enough energy from the environment. If the power demand of the connected electric application is known, a design space can be created in Fig. 9 showing all valid combinations of width and length satisfying application demands. This space is encircled by the solid green line which is constructed as follows: First, the power function Equation 2 is set to the necessary output power, e.g. $4mW$. In general, the solved equation has

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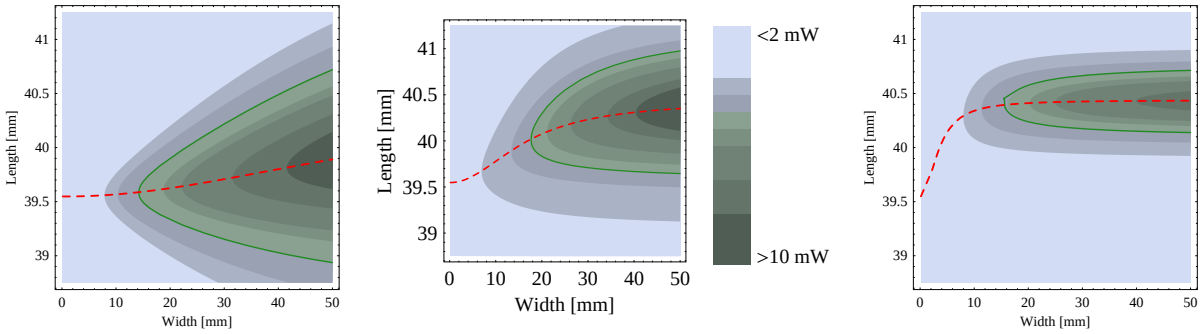
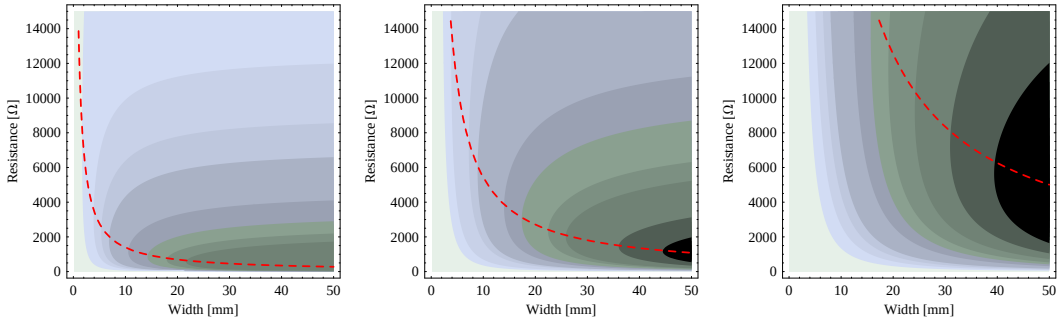


(a) $R_{Load} = 850\Omega$ (b) $R_{Load} = 3200\Omega$ (c) $R_{Load} = 15000\Omega$

Figure 9. Power Output depending on the bimorph's size for three different electric loads. The green areas show valid design spaces in which a total power drain of at least $4mW$ can be realised. The dashed lines show all combinations of the length and width where power output is optimal.



(a) $l = 39.6mm$ (b) $l = 40.0mm$ (c) $l = 40.4mm$

Figure 10. Power Output. Dependency of width and load resistance to the power drain over the resistor for different beam's length. The green areas show valid design spaces in which a total power drain of at least $4mW$ can be realised. The dashed lines show all combinations of the length and resistance where power output is optimal.

two solutions for each width at the minimum and the maximum length of the bimorph. This two points span the design space. All points within the bounding points are valid combinations.

The change of power output depending on the resistance is shown in Fig. 10. As can be seen the optimum resistance is falling with rising width of the bimorph due to the higher current output which affects the impedance of the piezoelectric element. This calculation held the element's length and height constant. The figure shows plots for different length of a piezoelectric bimorph. The length is varying between $39.6mm$ and $40.4mm$ according to the three different cases in Figure. 9. Again, it can be seen that the working frequency indicated by the length depends on the electric load.

In some cases the impedance of the electric application can be matched. This allows to alter the resistance in some extend. A helpful guide to select an adequate resistance is Fig. 10. Marking all combinations satisfying application needs shows the valid resistances that can be used regarding to the beam's width. The length of the beam will be determined as explained before using plots as shown in Fig. 9. If the resulting length yields a system with too less power output, an iteration of the design steps has to be done.

3.3. VALIDITY OF GEOMETRIC MODEL

The introduced geometric model allows the design of piezoelectric elements for an energy harvesting application in principle. Comparing the model using geometric properties with the model using identified parameter sets

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shows mostly the same behaviour. But there is one main difference: The real measured system shows no linear behaviour to the excitation level as assumed in he model. This results in higher damping factors if the system is excited on a higher level. Furthermore, an downwards frequency shift can be observed with higher excitation levels which is neither modelled in the model. The damping effect can be observed in Fig. 5.

However, this effects appears on higher excited systems. For low excitation levels the model gives a good approximation of the piezoelectric system's behaviour. The higher resonance frequency in the model using geometric and material properties may result due to the neglectance of the different stiffness of the substrate layer compared to the ceramics as well as due to a non ideal clamping of the bimorph during measurements.

4. CONCLUSION AND OUTLOOK

The paper shows a design method to obtain a suitable piezoelectric energy harvesting system which satisfies the power demands of an connected electronics. The design method utilises a modal model which can be analytically analysed. As briefly described, the modal model can be extended by geometric parameters to allow a system investigation regarding the dimensions of a pizeoelectric bimorph. The basic geometric equations are improved to fit the electric characteristics of the piezoelectric element.

The influence of the electric load is analysed in detail. Therefore, a procedure is shown to investigate the influence analytically. The analysis of the model shows that there are influences of the resistance on the working frequency to obtain an optimal power output which can basically be explained by the low current flow in antiresonance.

Utilising the possibility using geometrical data and keeping the change of the working frequency in mind allows the interpretation of the interdependence of the length and width of a bimorph relating to the load resistance. Resulting plots of different influences allows the construction of a valid design space. This design space basically contains all combinations of the length and width of the bimorph which satisfy application power demands. If necessary, a variation of the electric load can be included into the space.

Considering the problems of the utilisation of geometrical data for higher excitation levels shows the need for further research: Changes of damping and eigenfrequency (could be stiffness) due to non linear material properties which have to be included in the mathematical model. Another task is the high calculated resonance frequency. This may be improved by the modelling of the passive layer in the middle of the beam not only for the electric but also for the mechanic behaviour. Furthermore, the model analysis has to be validated by experiments.

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